TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 3 2019

Calculator Free

Proof

STUDENT'S NAME _____

DATE: Monday 20 May **TIME:** 50 minutes **MARKS**: 15

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Consider the statement: If n is prime, then n is odd.

(a) State the converse of the statement and comment on its validity. [2]

(b) State the inverse of the statement and comment on its validity. [2]

(c) State the contrapositive of the statement and comment on its validity. [2]

2. (5 marks)

For each of the following, decide whether the conjecture is true or false. If it is true, prove it. If it is false, give a counterexample.

(a) If two integers are multiplied then the result is always bigger than the two numbers. [2]

(b) When the product of two consecutive odd numbers is added to the next odd number, the result is always even. [3]

3. (3 marks)

For a group of 25 people, prove that there must be at least one day of the week on which at least 4 of those people were born. Justify your answer using the pigeon hole principle.

4. (3 marks)

From the set of counting numbers up to one hundred {1, 2, 3, 4, …, 99, 100}, a subset of k numbers is chosen. What is the smallest number that k can be, such that there must be a pair of numbers in the subset that add to an odd number? Justify your answer using the pigeon hole principle.

5. (4 marks)

A Diophantine equation is an equation for which only integer solutions are permitted.

Use the method of Proof by Contradiction to prove that there are no *positive* integer solutions to the Diophantine equations $x^2 - y^2 = 1$.

(That is, there are no *positive* combinations of *x* and *y* which satisfy this equation).

Hint: Factorise $x^2 - y^2 = 1$

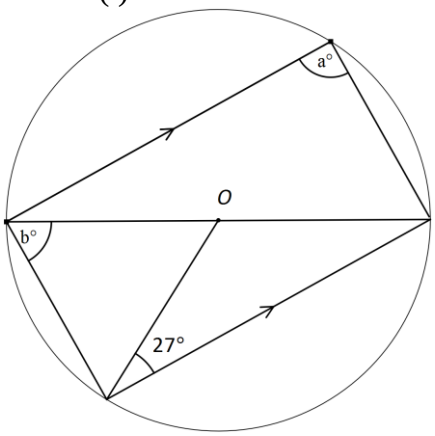
6. (5 marks)

Use proof by contradiction to prove that the opposite angles of a cyclic quadrilateral are supplementary.

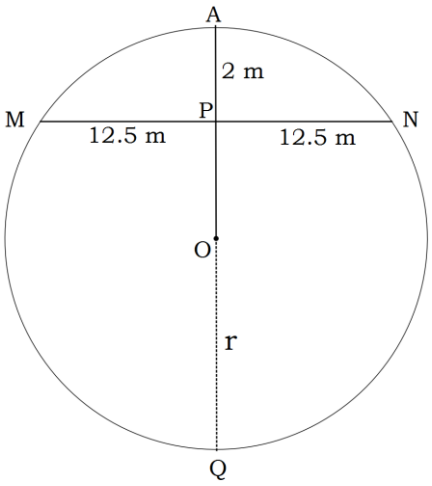
7. (9 marks)

(a) Find the value of the pronumeral in each of the following.

(i) [2]

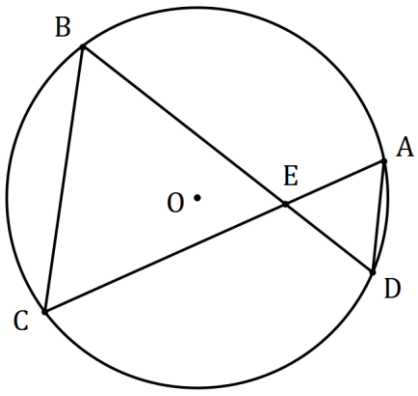


(ii) r = radius of the circle [2]



(b) In the circle with centre O drawn below, chord AC intersects chord BD at E.

(i) Explain, with reasoning, why triangles AED and BEC are similar. [3]

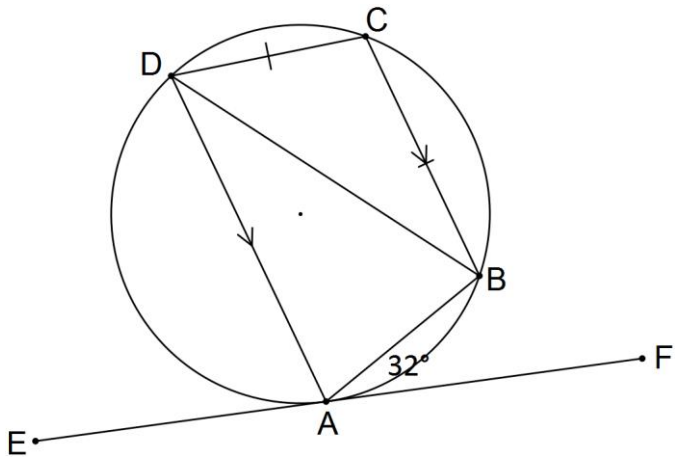


(ii) Prove that when two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord. [2]

8. (4 marks)

In the diagram below, ABCD is a cyclic quadrilateral.

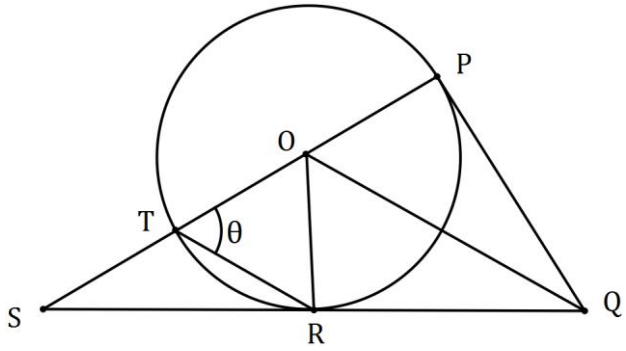
- EAF is a tangent to the circle at A.
- BC = CD.
- AD is parallel to BC.
- ∠BAF = 32°.



Determine, with **reasons**, the size of ∠BAD.

9. (10 marks)

In the diagram below, POT is a diameter of circle with centre O, QP is a tangent to the circle at P, QR is a tangent to the circle at R and PT is extended to meet QR extended at S. You may want to let $\angle OTR = \theta$.



(a) Prove that ΔOPQ is congruent to ΔORQ. [3]

(b) Prove that OQ is parallel to TR. [4]

(c) If $TR = TS$, deduce that ΔOTR is equilateral. [3]